Unpublished Supplement to Redistricting Principles and Racial Representation: A Reanalysis

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This unpublished supplement expands on the issues that were introduced in the article "Redistricting Principles and Racial Representation: A Reanalysis." It is divided into the following sections. First, the way in which state size effects how we should model the determinants of majority-minority (minority influence) districts is expanded on. Second, the formulas used to compute the maximum possible number of majority-minority (minority influence) districts are given and explained. Next, the various permutations of modeling decisions that appear in Tables 1 through 4 of this supplement are listed. Then the findings from these different modeling decisions are discussed, along with diagnostic tests and their implications for what should be concluded from the analyses. Last, the overall conclusions from the evidence presented here are summarized.

State Size

There are two issues that pertain to the size of states, in the analyses conducted in the Barabas and Jerit paper. The first is simply that the size of a state has a large impact on the number of majority-minority or minority influence districts a state has and must be controlled for. States with more districts, all else being equal, will have more majority-minority or minority influence districts. The large numbers of majority-minority districts some states have (California has 13, for example) cannot be observed in the majority of states (only eleven states have 13 or more districts). To fail to control for such an important determinant of majority-minority (minority influence) districts would threaten to introduce omitted variable bias (Kennedy 2003).

A second and different argument can be made that it is not enough to merely control for the size of states in the model. We would expect legal requirements to have a

larger impact on the number of districts in a state that is larger. Imagine two states, equal in all regards, except one has five congressional districts, and one has 50 congressional districts. Assume Section 5 of the VRA encourages the creation of majority-minority districts. It is obviously going to result in a greater increase in majority-minority districts in the larger state in comparison to the smaller state since there are ten times more districts there. Controlling for the number of districts in a state does not accurately model this interaction.

One way to model this relationship is to formally *interact* the number of districts in a state with every independent variable. In the *context* of a larger state, independent variables are expected to have a greater impact on the number of majority-minority districts. To keep the modeling strategy of the original paper—where the number of districts that are majority-minority is kept as the dependent variable—an interaction between every independent variable in the model and states' number of districts would need to be introduced. However, this modeling strategy would cause a great deal of multicollinearity. Because the a priori expectation that an independent variable has a greater impact on the number of majority-minority (minority influence) districts in large states is a strong one, this information can be brought into the model by dividing both the left and right hand side of the interactive model referred to above by "number of districts in a state." This keeps the independent variables as they are in the original model, but changes the dependent variable to a proportion.

Barabas and Jerit mention this problem in footnote 16 of their article, but imply that multicollinearity made controlling for the population of a state problematic. The present analysis did not find evidence of multicollinearity when "number of districts[1]" or

"state population^2" are added as control variables to their models one at a time. Furthermore, footnote 16 of their article states "In earlier analyses, we incorporated total population by examining the effects of redistricting standards on the percentage of majority-minority and minority influence districts in a state. We found similar results." If "similar results" means that all independent variables retain their status of achieving or not achieving statistical significance, similar results are not obtained from this strategy. Unlike in Barabas and Jerits' analysis, the variable "Compactness" is not statistically significant and negative in either the model examining majority-minority districts (model six of Table 2) or the model examining minority influence districts (model seventeen of Table 4). The findings of the reanalysis will be returned to below.

Formulas for Maximum Number of Districts

This section gives formulas that indicate how many possible majority-minority (minority influence) districts a state can have, and how these formulas are arrived at. Imagine that minorities are placed in a state in such a way as to maximize the number of majority-minority districts. This number of districts represents an absolute maximum number of majority-minority (minority influence) districts. This maximum is computed in the following way. First, the proportion of the state needed to create one majority-minority district is computed. This is the number one, divided by the number of districts in the state, multiplied by .5. We divide "1" by the number of districts to get the proportion of the state that lives in one district (assuming equal population at the beginning of the apportionment period, which is mandated by court rulings). We multiply by .5 because only half of a district needs to be a racial or ethnic minority for it

to be "majority-minority." The following formula indicates the proportion of the state we would need to fill up one-half of one district.

(1 / Number of Districts in State)*.5

To ascertain how many districts minorities could fill up and make majority-minority, if they are perfectly placed to do so, we divide the proportion of a state that is a racial or ethnic minority with the proportion of a state needed to create one such district. This is represented by the following formula.

Proportion Minority in State / [(1 / Number of Districts in State)*.5] = Absolute Limit on Number of Majority-Minority Seats

To compute the absolute limit on the number of minority influence districts in a state, the formula above merely needs to have the number ".5" replaced by the number ".35," since only 35 percent of a district needs to be a racial or ethnic minority for it to be a minority influence district.

The above formula indicates that the maximum number of majority-minority districts is not a simple function of the percent of minorities in a state. For example, Connecticut has 18.50 percent minorities, and no more than 20 percent of the districts in the state can be majority-minority as an absolute maximum (computed from the formula above). On the other hand, Tennessee, with 18.57 percent minorities has 33 percent of its districts majority-minority as an absolute maximum. What is the difference?

Connecticut has five districts, Tennessee has nine. If a state has only five districts, there must be a minimum of 10 percent minorities there before the possibility of having a majority-minority district. If there are ten districts, only five percent of the state need be minority for the possibility of having a majority-minority district.

The ten states with more than one district that cannot have majority-minority districts are Hawaii, Idaho, Iowa, Maine, Nebraska, New Hampshire, Oregon, Rhode Island, Utah, and West Virginia. Not surprisingly, these states are observed not to have majority-minority districts. Furthermore, no state with a score of 2.38 or lower from the above formula is observed to have any. Nine states cannot have minority-influence districts (the same as above, except Oregon).

Taking the states out of the analysis that cannot have majority-minority districts is equivalent to adding a dummy variable interaction with every independent variable, where the dummy variable is coded "1" if the state cannot possibly have a majority-minority (minority-influence) district. An analogous argument can be made to justify Barabas and Jerits' exclusion of the seven states that have only one district. We know with certainty that some states cannot have majority-minority (minority-influence) districts as a result of redistricting, and our analysis should reflect this.

Modeling Decisions

Good arguments can be made for several different modeling decisions. To reflect the fact that different scholars may favor some modeling decisions over others, the results of twenty-two models (eleven for each of Barabas and Jerit's two models) are displayed in this unpublished supplement. This also illustrates the robustness of some of the

differences in what is found in this reanalysis with the findings of Barabas and Jerit. Furthermore, it also allows the reader to see which modeling decisions result in divergent or consistent findings with Barabas and Jerit.

Tables 1 through 4 show the different models that are motivated by making several decisions one way or another. Tables 1 and 2 examine the determinants of majority-minority districts, while Tables 3 and 4 examine minority influence districts. The models in Tables 3 and 4 are the same as the corresponding models in Tables 1 and 2, but making the necessary changes so they are appropriate for analysis of minority influence districts. In the explanation of which models use which variables and cases below, parentheses indicate the number of the model with minority influence districts.

One modeling decision is whether to delete the states that cannot have majority-minority districts from analysis or not. It is clear that the states that cannot have majority-minority or minority influence districts should be excluded from analysis, so only a few models are shown with all the states that Barabas and Jerit examine. These models are merely to show the consequences of other modeling changes without dropping these cases.

Another decision is whether to keep the dependent variable as the number of majority-minority (minority influence) districts (Tables 1 and 3), or whether to examine it as a proportion of all districts in a state (Tables 2 and 4).

If the dependent variable is kept as the number of majority-minority (minority influence) districts, another modeling decision is whether to add the number of districts as a control variable, whether to add the number of possible majority-minority (minority influence) districts as a control variable, or whether to do neither. Model one of Table 1

(model twelve of Table 3) is Barabas and Jerits' original analysis of the determinants of the number of majority-minority (minority influence) districts. This is model one of Table 1 (model three of Table 1) in the published portion of this reanalysis. Model two of Table 1 (model thirteen of Table 3) merely drops the ten (nine) states that cannot have majority-minority (minority influence) districts. Model three (model fourteen) does not drop any cases, but adds a control for the number of districts a state has. Model four (model fifteen) goes back to dropping these ten states, and also adds a control for the number of districts a state has. Model five (model sixteen) controls for the maximum number of majority-minority districts a state can possibly have. In the few cases where the number of districts in a state is lower than this amount, it is the number of districts in a state.

Model six of Table 2 (model seventeen of Table 4) examines the determinants of the proportion of districts in a state that are majority-minority instead of the number of such districts. In all other regards, this model is the same as Barabas and Jerit's original model. Model seven of Table 2 (model eighteen of Table 4) drops the ten (nine) states that cannot have majority-minority (minority influence) districts from the analysis. Model eight (nineteen) adds a control for the maximum proportion of districts in a state that can possibly be majority-minority (minority influence) districts, while dropping the variable "Minority Population." Model nine (twenty) is the same as model eight (nineteen), except that "Minority Population" is added back as a control variable. It should be noted that this is the only time in which evidence of multicollinearity was uncovered in the analysis. Both the variable "Minority Population" and "Proportion of Districts Possible to be Majority-Minority" ("Proportion of Districts Possible to be

Minority Influence") displayed high levels of multicollinearity when regressing all of the other independent variables on them in turn, although this was not a problem for the other variables.

An alternative form of analysis to multiple regression, if the dependent variable is examined as a proportion, is Beta Regression. Beta Regression is specifically designed for analysis of dependent variables that vary between zero and one (Ferrari and Cribari-Neto 2004). On the other hand, Beta Regression is a fairly uncommon and new form of analysis, and lacking methodological studies to the contrary, the small sample properties of Beta Regression may be a cause for concern. SPSS was used to run a program for conducting Beta Regression.

Models ten and eleven of Table 2 (models twenty-one and twenty-two of Table 4) examine the results of Beta-Regressions. Model ten (twenty-one) is the same as Barabas and Jerit's original analysis, except that the dependent variable is the proportion of districts in a state that are majority-minority (minority influence). Model eleven (twenty-two) adds a control for "Proportion of Districts Possible to be Majority-Minority" to model ten (twenty-one).

Findings

Many modeling decisions do not matter for the findings, which makes explanation of the results strait-forward. Some broad generalizations can be made about the differences and consistencies between the new analysis and the original analysis of Barabas and Jerit. The findings for majority-minority districts are first discussed, followed by those of minority influence districts.

As long as the size of a state is somehow taken into account (whether controls for the number of districts or possible majority-minority districts are included, or whether the dependent variable is examined as a proportion), the variable "Compactness" fails to achieve statistical significance while negative (often by a large margin) (see models three through eleven of Tables 1 and all models of Table 2). When this coefficient stays negative, the t-value in these models never goes below -.07 (i.e., stays very close to zero). In all models of Table 2, the coefficient for "Compactness" takes on the opposite sign than that originally found (positive), although this never becomes statistically significant in any model.

Second, enforcement of section V of the Voting Rights Act is always associated with the creation of majority-minority districts in a statistically significance sense (see models one through eleven of Tables 1 and 2). This further strengthens the evidence found by Barabas and Jerit that Second V enforcement is associated with more majority-minority districts.

Third, as long as the size of a state is somehow taken into account (models three through eleven of Tables 1 and 2), the requirement that political subdivisions be protected when drawing district lines is marginally more statistically significant than in the model reported by Barabas and Jerit. These findings are essentially the same as in Barabas and Jerit, with the p-value merely moving from .14 in their model, to below .05 in models three through eleven of Tables 1 and 2 (two-tailed).

Like in Barabas and Jerit, unified Democratic control of state government was not associated with more majority-minority districts in any of the models in Tables 1 and 2. The other control variables ("Minority Population" and "Racial Segregation Index") also

behave much the same as in Barabas and Jerits' models, although "Racial Segregation Index" loses statistical significance by a large amount in several models (models nine, ten and eleven of Table 2).

The findings of the reanalyses of the determinants of minority influence districts are not as consistent across modeling decisions as those noted above. In analyses that examine the determinants of minority influence districts as a count, the analyses are substantively the same as Barabas and Jerit no matter what modeling decision is made (whether cases are dropped, or whether controls for the number of districts or possible minority influence districts are included).

More differences occur when minority influence districts are modeled as proportions. For all six models that examine minority influence districts as a proportion of all districts, "Compactness" does not attain statistical significance, unlike in Barabas and Jerit. The t-value goes from -2.66 in Barabas and Jerits' model to at most -.88 (in the sense of the largest absolute value) among the six models of Table 4. Again, convincing evidence that the compactness requirement is associated with minority districts is not uncovered.

Section V enforcement of the VRA is negatively related ($P<.05$) to the creation of minority influence districts in two out of four of the multiple regression models of Table 4. Barabas and Jerit found no such relationship. However, when Beta Regression (models twenty-one and twenty-two of Table 6) is utilized the new finding just mentioned is not corroborated. The presence of heteroskedasticity in the regression analyses of Table 4 further call into question the findings about Section V VRA

enforcement from the regression analysis (see below). Therefore, evidence was not found that Barabas and Jerit's findings about the VRA are in error.

Political subdivisions obtain the same relationship to minority influence districts that Barabas and Jerit found, and this variable is statistically significant in all six models of Table 4. In all models of Table 4, the control variables ("Unified Democratic Control," "Minority Population," and "Racial Segregation Index") behave much the same as in Barabas and Jerit's analysis.

Diagnostic tests indicate that when regression is used to analyze proportions the assumptions of multiple regression are not violated for majority-minority districts (Table 2), but there is evidence of heteroskedasticity for the analyses of minority influence districts (Table 4). A simple test is to see how many out of bounds predictions the regression models yield. For the models that delete the ten states that cannot have majority-minority districts, only between zero to four of the 33 states have negative predicted values (models six to nine, Table 2). The analyses of minority influence districts are more problematic, with between five and ten negative predicted values for 34 states (models seventeen to twenty of Table 4).

The Breusch-Pagan / Cook-Weisberg test for heteroskedasticity (the STATA command "hetttest") indicates no evidence of heteroskedasticity for the analysis of majority-minority districts. However, this test indicated strong evidence of heteroskedasticity for all analyses of minority influence districts (Table 4, models seventeen through twenty). To deal with this problem, robust standard errors were computed for the regression analyses. The presence of heteroskedasticity in these models indicates that more credence should be given to the results of the Beta-Regression

analysis of minority influence districts when the findings of models seventeen through twenty of Table 4 diverge with those of models twenty-one and twenty-two of Table 4. This means the evidence that Section 5 of the VRA is associated with fewer minority influence districts not less compelling, and does not contradict Barabas and Jerits' findings.

If robust standard errors are not computed, for all the regression models of Tables 2 and 4, the only substantive difference (i.e., effecting whether a variable attains statistical significance or not) is that Section V enforcement of the VRA is statistically significant ($p<.05$) in two models where it was not before: model seventeen of Table 4, and model nineteen of Table 4.

Conclusion

A strong case can be made that examining majority-minority and minority influence districts as proportions is the correct modeling strategy, and it is argued here that the findings from those analyses are the most telling. This implies that the compactness requirement does not cause the creation of fewer majority-minority or minority influence districts. The other four main findings of Barabas and Jerit are much the same as in this analysis. "Voting Rights Act § 5" is associated with fewer majority-minority districts, as in Barabas and Jerit. "Political Subdivisions" is related to fewer majority-minority districts as in Barabas and Jerits' analysis, although it narrowly misses statistical significance there. Barabas and Jerit found a null relationship between "Voting Rights Act § 5" and minority influence districts. Good evidence was not uncovered that overturned this null finding. Last, the protection of political subdivisions in redistricting was found to be associated with the creation of more minority influence districts, as Barabas and Jerit found. In summary,

the two most prominently mentioned findings of Barabas and Jerit do not hold in this analysis: the compactness requirement does not matter for the creation of the two types of minority districts.

Table 1: Determinants of Number of Majority-Minority Districts: Poisson Analyses

	Model one	Model two	Model three	Model four	Model five
Compactness	-.796* (.361) -2.21	-.686 (.370) -1.85	-.027 (.389) -.07	.232 (.413) 0.56	-.016 (.351) -.04
Voting Rights Act § 5	1.448* (.460) 3.15	1.435* (.433) 3.31	.892* (.443) 2.01	.870* (.405) 2.15	1.060* (.438) 2.42
Political Subdivisions	-.436 (.293) -1.49	-.452 (.321) -1.41	-.777* (.249) -3.12	-.885* (.288) -3.07	-.796* (.258) -3.09
Unified Democratic Control	.453 (.356) 1.27	.498 (.377) 1.32	-.001 (.272) -0.00	.109 (.259) 0.42	.164 (.299) 0.55
Minority Population	6.413* (1.490) 4.30	5.748* (1.477) 3.89	4.255* (1.435) 2.96	3.451* (1.330) 2.59	2.220 (1.779) 1.25
Racial Segregation Index	4.410* (1.610) 2.74	4.506* (1.607) 2.80	.853 (1.366) 0.63	.965 (1.311) 0.74	1.550 (1.543) 1.00
Number of Districts	_____	_____	.048* (.014) 3.55	.050* (.014) 3.58	_____
Number of Possible Majority-Minority Districts	_____	_____	_____	_____	.052* (.016) 3.18
Constant	-4.784* (1.297) -3.69	-4.612* (1.308) -3.52	-2.285* (.978) -2.34	-2.123* (.927) -2.29	-1.932 (1.276) -1.51
Log-Likelihood	-44.700	-42.717	-39.552	-37.261	-39.218
N	43	33	43	33	33

The cell entries are Poisson Regression coefficients, with robust standard errors in parentheses. *=p<.05. All tests of statistical significance are two-tailed.

The dependent variable for models one through five is the number of majority-minority districts that a state has.

Table 2: Determinants of Majority-Minority Districts as Proportions

	Model six	Model seven	Model eight	Model nine	Model ten	Model eleven
Compactness	.004 (.020) 0.21	.030 (.021) 1.42	.035 (.020) 1.76	.040 (.023) 1.79	.530 (.525) 1.01	.618 (.527) 1.17
Voting Rights Act § 5	.106* (.034) 3.13	.126* (.036) 3.50	.097* (.039) 2.50	.074* (.031) 2.38	1.716* (.637) 2.69	1.467* (.673) 2.18
Political Subdivisions	-.058* (.019) -2.97	-.104* (.024) -4.43	-.090* (.024) -3.76	-.075* (.023) -3.33	-1.127* (.558) -2.02	-1.049 (.566) -1.85
Unified Democratic Control	.020 (.023) 0.86	.041 (.032) 1.30	.025 (.027) 0.91	.013 (.025) 0.52	.511 (.484) 1.06	.474 (.488) .97
Minority Population	.481* (.148) 3.26	.522* (.169) 3.08	_____	-.740* (.336) -2.20	4.393 (2.859) 1.54	_____
Racial Segregation Index	.109 (.072) 1.52	.171* (.083) 2.06	.129 (.070) 1.85	.063 (.070) 0.89	1.389 (1.561) .89	1.149 (1.600) .72
Proportion of Districts Possible to be Majority-Minority	_____	_____	.335* (.087) 3.86	.727* (.165) 4.41	_____	2.774 (1.500) 1.85
Constant	-.096 (.056) -1.71	-.150* (.067) -2.25	-.128* (.048) -2.68	-.065 (.058) -1.12	-5.015* (1.308) -3.83	-4.903* (1.254) -3.91
Standard Error of the Estimate	.056	.057	.050	.046	_____	_____
D0 parameter	_____	_____	_____	_____	-2.594* (.279) -9.30	-2.671* (.290) -9.21
R-Squared	.758	.769	.828	.856	_____	_____
N	43	33	33	33	33	33

The cell entries for models six through nine are unstandardized Regression coefficients, with robust standard errors in parentheses. The cell entries for models ten and eleven are

Beta Regression coefficients with bootstrap standard errors in parentheses. *=p<.05. All tests of statistical significance are two-tailed.
The dependent variable is the proportion of all districts in a state that are majority-minority.

Table 3: Determinants of Number of Minority Influence Districts: Poisson Analyses

	Model twelve	Model thirteen	Model fourteen	Model fifteen	Model sixteen
Compactness	-1.508* (.566) -2.66	-1.450* (.550) -2.63	-1.126* (.470) -2.40	-1.047* (.453) -2.31	-1.028* (.454) -2.27
Voting Rights Act § 5	.064 (.472) 0.14	.083 (.455) 0.18	-.341 (.403) -0.85	-.327 (.392) -0.83	-.339 (.396) -0.86
Political Subdivisions	1.270* (.388) 3.27	1.268* (.392) 3.23	1.144* (.354) 3.23	1.147* (.360) 3.19	1.116* (.360) 3.10
Unified Democratic Control	.136 (.353) 0.39	.144 (.358) 0.40	-.295 (.441) -0.67	-.294 (.442) -0.67	-.328 (.460) -0.71
Minority Population	12.539* (1.694) 7.40	12.057* (1.830) 6.59	10.859* (1.689) 6.43	10.228* (1.858) 5.51	9.524* (1.968) 4.84
Racial Segregation Index	2.352 (1.515) 1.55	2.414 (1.509) 1.60	-.445 (1.612) -0.28	-.404 (1.552) -0.26	-.452 (1.515) -0.30
Number of Districts	_____	_____	.031* (.014) 2.16	.032* (.014) 2.23	_____
Number of Possible Majority-Minority Districts	_____	_____	_____	_____	.033* (.015) 2.21
Constant	-5.630* (1.417) -3.97	-5.516* (1.430) -3.86	-3.506* (1.221) -2.87	-3.342* (1.214) -2.75	-3.012* (1.281) -2.35
Log-Likelihood	-31.545	-31.177	-29.758	-29.298	-29.216
N	43	34	43	34	34

The cell entries are Poisson Regression coefficients, with robust standard errors in parentheses. *=p<.05. All tests of statistical significance are two-tailed.

The dependent variable for models fourteen through eighteen is the number of minority influence districts that a state has.

Table 4: Determinants of Minority Influence Districts as Proportions

	Model seventeen	Model eighteen	Model nineteen	Model twenty	Model twenty-one	Model twenty-two
Compactness	-.004 (.047) -0.08	-.042 (.052) -0.82	-.048 (.054) -0.88	-.034 (.053) -.064	-.407 (.693) -.59	-.380 (.684) -.56
Voting Rights Act § 5	-.170 (.098) -1.74	-.211* (.101) -2.08	-.224 (.120) -1.86	-.184* (.087) -2.12	-1.686 (.877) -1.92	-1.746 (1.013) -1.72
Political Subdivisions	.139* (.051) 2.73	.198* (.065) 3.02	.221* (.077) 2.89	.169* (.063) 2.69	1.830* (.762) 2.40	1.882* (.802) 2.35
Unified Democratic Control	-.027 (.057) -0.47	-.092 (.075) -1.23	-.106 (.082) -1.29	-.068 (.066) -1.02	-1.224 (.814) -1.50	-1.284 (.829) -1.55
Minority Population	1.219* (.488) 2.50	1.538* (.536) 2.87	_____	3.058* (1.692) 1.81	14.173* (5.090) 2.78	_____
Racial Segregation Index	-.072 (.126) -0.57	-.031 (.148) -0.21	-.066 (.160) -0.41	-.013 (.145) -0.09	-1.294 (1.629) -.79	-1.581 (1.727) -.92
Proportion of Districts Possible to be Majority-Minority	_____	_____	.564* (.223) 2.53	-.611 (.545) -1.12	_____	4.827* (2.015) 2.40
Constant	-.118 (.093) -1.26	-.207 (.134) -1.55	-.159 (.134) -1.19	-.219 (.137) -1.60	-4.860* (1.707) -2.85	-3.938* (1.809) -2.18
Standard Error of the Estimate	.128	.131	.140	.129	_____	_____
R-Squared	.527	.610	.549	.631	_____	_____
D0 parameter	_____	_____	_____	_____	-1.827* (.490) -3.73	-1.443* (.527) -2.74
N	43	34	34	34	34	34

The cell entries for models seventeen through twenty are unstandardized Regression coefficients, with robust standard errors in parentheses. The cell entries for models twenty-one and twenty-two are Beta Regression coefficients with bootstrap standard errors in parentheses. *=p<.05. All tests of statistical significance are two-tailed. The dependent variable is the proportion of all districts in a state that are minority influence.

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[1] Data on the number of districts a state had were obtained from Bureau of the Census (2004, 219).

[2] Obtained from http://www.bea.gov/bea/regional/spi/default.cfm?satable=summary accessed July 25, 2006.